Solving covariance function models with MiX99

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MiX99 course: test-day models and single step genomic prediction

COURSE DAY 2, April 11th, 2025

Aim

Demonstrate the solving of covariance function models with MiX99 using 2 examples

- 2 Traits: repeated observations from 1st parity milk yield and feed intake
- 9 Traits: repeated test-day observations from 1st, 2nd, and 3rd parity milk, protein, and fat yield

Example 1: Covariance function model for describing longitudinal milk yield and feed intake data

Describing milk and intake by a random regression model

- Let's consider the following model:

$$milk_{ijklm} = f_1(cage) + f_{1:c}(dim) + HTM_{1:h} + f_{1:n}(dim) + f_{1:a}(dim) + e_{1:ijklm}$$
$$intake_{ijklm} = f_2(cage) + f_{2:c}(dim) + HTM_{2:h} + f_{2:n}(dim) + f_{2:a}(dim) + e_{2:ijklm}$$

where:

i=calving age, j=herd, k=contemporary group, l=animal, m=days in milk

Fixed regression functions for calving age and herd-specific lactation curves

$$f_1(cage) = cage \times b_{1:1} + cage^2 \times b_{1:2}$$

$$f_2(cage) = cage \times b_{2:1} + cage^2 \times b_{2:2}$$

$$f_{1:c}(dim) = \sum_{s=0}^{3} \phi_{sm} c_{1:smj} + e^{-0.05\times m} c_{1:4mj}$$

$$f_{2:c}(dim) = \sum_{s=0}^{3} \phi_{sm} c_{2:smj} + e^{-0.05\times m} c_{2:4mj}$$

3rd order Legendre

Random regression functions for nonhereditary (i.e. PE) and additive genetic animal effects

$$f_{1:n}(dim) = \sum_{s=0}^{2} \phi_{sm} n_{1:sml}$$

$$f_{2:n}(dim) = \sum_{s=0}^{2} \phi_{sm} n_{2:sml}$$

$$f_{1:a}(dim) = \sum_{s=0}^{2} \phi_{sm} a_{1:sml}$$

$$f_{2:a}(dim) = \sum_{s=0}^{2} \phi_{sm} a_{2:sml}$$

2nd order Legendre

Describing milk and intake by a random regression model

Data file

```
ANI  HERD  HTD       DIM  cage1   cage2   milk  dmi
867 1 12000032 262 0.2822 0.0796 26.1 19.0
867 1 12000032 269 0.2822 0.0796 25.5 19.4
100 1 11999021  10 0.0795 0.0063 22.2 12.7
100 1 11999021  17 0.0795 0.0063 27.3 17.8
100 1 11999021  24 0.0795 0.0063 29.5 19.1
```

Table file

```
     t1                  t2                   t3                  t4                   t5
DIM  Leg_cov0            Leg_cov1             Leg_cov2            Leg_cov3             exp(-0.005*DIM)
5 0.7071067811865476 -1.224744871391589 1.58113883008419 -1.870828693386971 0.7788007830714049
6 0.7071067811865476 -1.217940733217191 1.554859717121216 -1.80889999717125 0.7408182206817179
7 0.7071067811865476 -1.211136595042793 1.528727005901769 -1.74783261354842 0.7046880897187134
8 0.7071067811865476 -1.204332456868396 1.502740696425848 -1.687621730716286 0.670320046035639
```

CLIM file

```
#-----------------------------------------------------------------------------------------------
  TITLE     Random regression model for milk yield and dry matter intake

  DATAFILE  /homeappl/home/ejo31/MiX99_course_Italy2025/random_regression_FEdata/data/weekly_FE_FIN062022.dat
  PEDFILE   /homeappl/home/ejo31/MiX99_course_Italy2025/random_regression_FEdata/data/pedigree.FIN062022.G24
  PARFILE   /homeappl/home/ejo31/MiX99_course_Italy2025/random_regression_FEdata/data/parfile

  INTEGER    ANI  HERD  HTM  DIM
  REAL       cage1  cage2  milk  dmi

  DATASORT  BLOCK=HERD PEDIGREECODE=ANI
  MISSING   -9999.0

  MODEL
  milk =  cage1 cage2 LACCRV(t1 t2 t3 t4 t5|HERD)  HTM  NH(t1 t2 t3|ANI)  G(t1 t2 t3|ANI)
  dmi  =  cage1 cage2 LACCRV(t1 t2 t3 t4 t5|HERD)  HTM  NH(t1 t2 t3|ANI)  G(t1 t2 t3|ANI)

  RANDOM   NH G

  PEDIGREE  G  am

  WITHINBLOCKORDER G PE HTM
  PRECON           b  b  b     b  # last for other fixed effects

  TABLEFILE   /homeappl/home/ejo31/MiX99_course_Italy2025/random_regression_FEdata/data/3rdOrd.LegPolWil005.cov
  TABLEINDEX  DIM
  TMPDIR  tmpMiX
#-----------------------------------------------------------------------------------------------
```

G(t1 t2 t3|ANI): milk 1, 3, 5; dmi 2, 4, 6

Order of:
- variances in VCV PARFILE
- random regression coefficient estimates by factors and within factor by traits

VC parameter file

```
1   1   1   11.045367
1   2   1    2.6545205
1   2   2    4.6279004
1   3   1    1.2888027
1   3   2   0.87167377
1   3   3    6.0185870
1   4   1    1.9401911
1   4   2    3.4300993
1   4   3    2.2856570
1   4   4    5.7003425
1   5   1   0.91984633
1   5   2   0.68534811
1   5   3    2.7989419
1   5   4   0.92106420
1   5   5    2.0720462
1   6   1   0.89351743
1   6   2    1.2996433
1   6   3    1.1220068
1   6   4    2.8485769
1   6   5   0.77265012
1   6   6    2.1694068
2   1   1    9.7868457
2   2   1    4.3306125
2   2   2    2.7686328
2   3   1   0.51572484
2   3   2   0.43839421
2   3   3    2.3333322
2   4   1  -0.42222210
2   4   2  -0.27452962
```

Building covariance functions for both animal effects

- Our original VCV matrices for nonhereditary and additive genetic animal effects are:

- $\mathbf{K}_n$ =

11.04537	1.28880	0.91985	2.65452	1.94019	0.89352
1.28880	6.01859	2.79894	0.87167	2.28566	1.12201
0.91985	2.79894	2.07205	0.68535	0.92106	0.77265
2.65452	0.87167	0.68535	4.62790	3.43010	1.29964
1.94019	2.28566	0.92106	3.43010	5.70034	2.84858
0.89352	1.12201	0.77265	1.29964	2.84858	2.16941

- $\mathbf{K}_a$ =

9.786846	0.515725	-1.114154	4.330612	-0.422222	-1.528051
0.515725	2.333332	0.433607	0.438394	0.045220	-0.084024
-1.114154	0.433607	1.197352	-0.355226	0.033419	-0.027669
4.330612	0.438394	-0.355226	2.768633	-0.274530	-0.826908
-0.422222	0.045220	0.033419	-0.274530	0.454212	0.144860
-1.528051	-0.084024	-0.027669	-0.826908	0.144860	0.483692

- Let's go through the derivation of covariance functions for the nonhereditary animal effects extracting the correlation matrix…

```
octave:3> Kn
Kn =

   11.04537    1.28880    0.91985    2.65452    1.94019    0.89352
    1.28880    6.01859    2.79894    0.87167    2.28566    1.12201
    0.91985    2.79894    2.07205    0.68535    0.92106    0.77265
    2.65452    0.87167    0.68535    4.62790    3.43010    1.29964
    1.94019    2.28566    0.92106    3.43010    5.70034    2.84858
    0.89352    1.12201    0.77265    1.29964    2.84858    2.16941

octave:4> S=diag(sqrt(diag(Kn)))
S =

Diagonal Matrix

   3.3235        0        0        0        0        0
        0   2.4533        0        0        0        0
        0        0   1.4395        0        0        0
        0        0        0   2.1513        0        0
        0        0        0        0   2.3875        0
        0        0        0        0        0   1.4729

octave:5> Cn=inv(S)*Kn*inv(S)
Cn =

   1.00000   0.15807   0.19228   0.37128   0.24451   0.18253
   0.15807   1.00000   0.79259   0.16516   0.39022   0.31051
   0.19228   0.79259   1.00000   0.22132   0.26800   0.36443
   0.37128   0.16516   0.22132   1.00000   0.66783   0.41017
   0.24451   0.39022   0.26800   0.66783   1.00000   0.81004
   0.18253   0.31051   0.36443   0.41017   0.81004   1.00000

octave:6>
```

original $\mathbf{K}_n$ matrix

correlation matrix

decomposing the correlation matrix...

```
octave:6> [V D]=eig(Cn)
V =

   0.257104   0.189826   0.853861   0.405321   0.063423   0.021847
   0.394104  -0.578536   0.019864  -0.090952   0.585111  -0.398694
   0.392209  -0.576084   0.090749  -0.099473  -0.586256   0.390479
   0.401716   0.410788   0.154346  -0.728829  -0.179025  -0.287771
   0.501600   0.303797  -0.282210   0.035312   0.411645   0.636995
   0.460188   0.190658  -0.398529   0.533956  -0.329280  -0.446674

D =

Diagonal Matrix

   2.919486          0          0          0          0          0
          0   1.312845          0          0          0          0
          0          0   0.925216          0          0          0
          0          0          0   0.511485          0          0
          0          0          0          0   0.272253          0
          0          0          0          0          0   0.058713

octave:7> [s,i]=sort(diag(D));
octave:8> D=diag(s)
D =

Diagonal Matrix

   0.058713          0          0          0          0          0
          0   0.272253          0          0          0          0
          0          0   0.511485          0          0          0
          0          0          0   0.925216          0          0
          0          0          0          0   1.312845          0
          0          0          0          0          0   2.919486

octave:9> V=V(:,i(:))
V =

   0.021847   0.063423   0.405321   0.853861   0.189826   0.257104
  -0.398694   0.585111  -0.090952   0.019864  -0.578536   0.394104
   0.390479  -0.586256  -0.099473   0.090749  -0.576084   0.392209
  -0.287771  -0.179025  -0.728829   0.154346   0.410788   0.401716
   0.636995   0.411645   0.035312  -0.282210   0.303797   0.501600
  -0.446674  -0.329280   0.533956  -0.398529   0.190658   0.460188
```

sorted eigenvalues and eigenvectors

keeping only the largest eigenvalues that explain at least a minimum required amount of total variance...


```
octave:17> TotalVar=sum(s)
TotalVar =  6.0000
octave:18> Eigenvalues_in_percent=(s./TotalVar)*100
Eigenvalues_in_percent =

    0.97856
    4.53755
    8.52475
   15.42027
   21.88076
   48.65811

octave:19> cumVar=0.0; rank=0; i=0;
octave:20> N=6
N =  6
octave:21> while (cumVar<99.0)
> cumVar=cumVar+Eigenvalues_in_percent(N-i);
> rank=rank+1;
> i=i+1;
> endwhile
octave:22> rank
rank =  5
octave:23>  d=N-rank; Deleted_Eigenvalues=N-rank
Deleted_Eigenvalues =  1
octave:24> remainingVar_in_percent=(sum(diag(D(d+1:N,d+1:N)))/TotalVar)*100
remainingVar_in_percent =  99.021
octave:25> Vx=V(:,Deleted_Eigenvalues+1:N)
Vx =

   0.063423   0.405321   0.853861   0.189826   0.257104
   0.585111  -0.090952   0.019864  -0.578536   0.394104
  -0.586256  -0.099473   0.090749  -0.576084   0.392209
  -0.179025  -0.728829   0.154346   0.410788   0.401716
   0.411645   0.035312  -0.282210   0.303797   0.501600
  -0.329280   0.533956  -0.398529   0.190658   0.460188

octave:26> Dx=D(Deleted_Eigenvalues+1:N,Deleted_Eigenvalues+1:N)
Dx =

   0.27225   0.00000   0.00000   0.00000   0.00000
   0.00000   0.51149   0.00000   0.00000   0.00000
   0.00000   0.00000   0.92522   0.00000   0.00000
   0.00000   0.00000   0.00000   1.31285   0.00000
   0.00000   0.00000   0.00000   0.00000   2.91949
```

let's require that at least 99% of the total variance will be explained

retained eigenvectors and eigenvalues

expressing the coefficient matrix $\mathbf{K}_n$ by $\mathbf{E}^*\mathbf{E}^{*\prime}$…

```
octave:27> Vxd=Vx*sqrt(Dx)
Vxd =

   0.033093   0.289878   0.821314   0.217502   0.439300
   0.305298  -0.065047   0.019107  -0.662883   0.673387
  -0.305896  -0.071141   0.087290  -0.660074   0.670148
  -0.093411  -0.521246   0.148462   0.470679   0.686393
   0.214788   0.025254  -0.271453   0.348089   0.857060
  -0.171811   0.381875  -0.383338   0.218454   0.786300

octave:28> disp("difference rank-reduced minus original correlation matrix (in %):")
difference rank-reduced minus original correlation matrix (in %):
octave:29> Cx=Vxd*Vxd';
octave:30> diff=(Cx-Cn)*100
diff =

  -0.0028024   0.0511411  -0.0500873   0.0369128  -0.0817084   0.0572956
   0.0511411  -0.9332890   0.9140590  -0.6736331   1.4911206  -1.0456050
  -0.0500873   0.9140590  -0.8952253   0.6597532  -1.4603968   1.0240609
   0.0369128  -0.6736331   0.6597532  -0.4862176   1.0762671  -0.7547010
  -0.0817084   1.4911206  -1.4603968   1.0762671  -2.3823712   1.6705686
   0.0572956  -1.0456050   1.0240609  -0.7547010   1.6705686  -1.1714377

octave:31> Ex=S*Vxd
Ex =

   0.109982   0.963397   2.729601   0.722857   1.459995
   0.748983  -0.159579   0.046875  -1.626239   1.652007
  -0.440325  -0.102405   0.125650  -0.950151   0.964652
  -0.200952  -1.121333   0.319381   1.012551   1.476607
   0.512814   0.060296  -0.648105   0.831077   2.046264
  -0.253059   0.562461  -0.564615   0.321760   1.158135

octave:32> Kn_x=Ex*Ex'
Kn_x =

   11.04506    1.29297    0.91745    2.65716    1.93371    0.89632
    1.29297    5.96242    2.83122    0.83612    2.37300    1.08423
    0.91745    2.83122    2.05350    0.70578    0.87087    0.79436
    2.65716    0.83612    0.70578    4.60540    3.48538    1.27573
    1.93371    2.37300    0.87087    3.48538    5.56454    2.90733
    0.89632    1.08423    0.79436    1.27573    2.90733    2.14400
```

difference (in %)
original – rank-reduced correlation matrix

rank-reduced $\mathbf{K}_n$ matrix

deriving the final covariate matrix **U**…

```
octave:42> load TableFile
octave:43> TableFile(1:5,:)
ans =

   5.00000   0.70711  -1.22474   1.58114  -1.87083   0.77880
   6.00000   0.70711  -1.21794   1.55486  -1.80890   0.74082
   7.00000   0.70711  -1.21114   1.52873  -1.74783   0.70469
   8.00000   0.70711  -1.20433   1.50274  -1.68762   0.67032
   9.00000   0.70711  -1.19753   1.47690  -1.62826   0.63763

octave:44> PHI0=TableFile(:,2:4);
octave:45> PHI0(1:5,:)
ans =

   0.70711  -1.22474   1.58114
   0.70711  -1.21794   1.55486
   0.70711  -1.21114   1.52873
   0.70711  -1.20433   1.50274
   0.70711  -1.19753   1.47690

octave:46> I2=eye(2)
I2 =

Diagonal Matrix

   1   0
   0   1

octave:47> PHI=kron(I2,PHI0);
octave:48> PHI(1:5,:)
ans =

   0.70711  -1.22474   1.58114   0.00000   0.00000   0.00000
   0.70711  -1.21794   1.55486   0.00000   0.00000   0.00000
   0.70711  -1.21114   1.52873   0.00000   0.00000   0.00000
   0.70711  -1.20433   1.50274   0.00000   0.00000   0.00000
   0.70711  -1.19753   1.47690   0.00000   0.00000   0.00000

octave:49> U=PHI*Ex;
octave:50> U(1:5,:)
ans =

  -1.53576   0.71475   2.07138   1.00055   0.53433
  -1.51909   0.71636   2.06840   1.01445   0.52022
  -1.50249   0.71795   2.06543   1.02821   0.50626
  -1.48595   0.71952   2.06249   1.04184   0.49243
  -1.46948   0.72108   2.05956   1.05533   0.47874
```

by multiplying $\mathbf{\Phi}$ with the scaled eigenfunction matrix $\mathbf{E}^*$, we get the covariate matrix (**U**) of the covariance function

please note, here we get 5 test-day × trait-specific covariates

Modelling milk and intake by covariance functions

Data file

ANI HERD HTD DIM cage1 cage2 milk dmi

```
867 1 12000032 262 0.2822 0.0796 26.1 19.0
867 1 12000032 269 0.2822 0.0796 25.5 19.4
100 1 11999021  10 0.0795 0.0063 22.2 12.7
100 1 11999021  17 0.0795 0.0063 27.3 17.8
100 1 11999021  24 0.0795 0.0063 29.5 19.1
```

Table file

DIM t1 t2 t3 t4 t5 t6 … t27

```
 5 0.70710678118654757 -1.2247448713915889 1.58113883008419 -1.8708286933869711 0.77880078307140488 -1.5357520
253265984 0.71475639493439613 2.0713724602121228 1.0005478998515305 0.53433280326203514 -1.1702863034586846 0.
022578979115052075 0.12686197253855439 0.20687353304706158 0.36913812711002336 -0.79634079943321945 -0.61455565
813982249 -1.747709438409645 -0.65492563963763328 -0.26511636184340387 -1.3256409835656648 0.16903120551248663
 -0.72165829094152878 0.62550552099738721 -0.36624840466025149 -0.18414419548101166 -0.48128537593400728
 6 0.70710678118654757 -1.217940733217191 1.554859717121216 -1.8088999971712501 0.74081822068171788 -1.5190845
```

CLIM file

```
#-----------------------------------------------------------------------------------------------------------
  TITLE     Covariance function model for milk yield and dry matter intake

  DATAFILE  /homeappl/home/ejo31/MiX99_course_Italy2025/random_regression_FEdata/data/weekly_FE_FIN062022.dat
  PEDFILE   /homeappl/home/ejo31/MiX99_course_Italy2025/random_regression_FEdata/data/pedigree.FIN062022.G24
  PARFILE   /homeappl/home/ejo31/MiX99_course_Italy2025/martin/MTCF/parfile_reduced.txt

  INTEGER    ANI  HERD  HTM  DIM
  REAL       cage1  cage2  milk  dmi

  DATASORT  BLOCK=HERD PEDIGREECODE=ANI
  MISSING   -9999.0

  MODEL
  milk =  cage1 cage2 LACCRV(t1 t2 t3 t4 t5|HERD)  HTM  PE( t6:10|ANI)@1  G(t16:21|ANI)@1
  dmi  =  cage1 cage2 LACCRV(t1 t2 t3 t4 t5|HERD)  HTM  PE(t11:15|ANI)@1  G(t22:27|ANI)@1

  RANDOM   NH G

  PEDIGREE  G  am

  WITHINBLOCKORDER G PE HTM
  PRECON            b  b  b     b  # latter for other fixed effects

  TABLEFILE   /homeappl/home/ejo31/MiX99_course_Italy2025/martin/MTCF/CovariableTable.txt
  TABLEINDEX  DIM
  TMPDIR  tmpMiX
#-----------------------------------------------------------------------------------------------------------
```

1 2 3 4 5

1 2 3 4 5 6

Order of:
- variances in VCV PARFILE
- covariance function coefficient estimates by factors

additive genetic effect

Combining effects across traits

VC parameter file

```
1  1  1  1.000000
1  2  2  1.000000
1  3  3  1.000000
1  4  4  1.000000
1  5  5  1.000000
2  1  1  1.000000
2  2  2  1.000000
2  3  3  1.000000
2  4  4  1.000000
2  5  5  1.000000
2  6  6  1.000000
3  1  1  2.250144
3  2  1  0.537894
3  2  2  1.327405
```

Example 2: Covariance function model with reduced number of parameters for a multi-trait test-day model

Example data and model

(Lidauer et al., 2015, JDS)

Data

- 20k cows with records, 30k animals in pedigree
- 353k test-day records

Model

- Multi-trait, 9 traits: milk, protein, fat in 1st, 2nd, and 3rd parity
- Fixed effects
  - lactation curves nested in calving 2-years-periods, calving age, days carried calf, herd-2-years-period
- Random effects
  - herd-test-day
  - herd curves nested within calving 2-years-periods (RR: 3 coefficients /trait)
  - nonhereditary animal effect (RR: 4 coefficients /trait)
  - additive genetic animal effect (RR: 4 coefficients /trait)
  - random residuals: 12 lactation period classes

  → in total, 2340 variance component parameters

Multi-trait random regression model

Data file

ANI	HRD	TGRP	HTM	HC2Y	C2Y	AGE	DCC	DRY	DIM	RC	milk	protein	fat
2669	464	2	128223	266903	3	6	1	3	309	11	10.9	8.0	6.8
2669	464	2	128224	266903	3	6	1	3	335	12	9.2	7.0	6.0
2669	464	3	128215	266904	4	7	1	4	33	2	20.7	12.4	9.9
2669	464	3	128216	266904	4	7	1	4	73	5	17.9	10.4	8.6

Table file

DIM	t1 Leg_cov0	t2 Leg_cov1	t3 Leg_cov2	t4 Leg_cov3	t5 exp(-0.005*DIM)
5	0.7071067811865476	-1.224744871391589	1.58113883008419	-1.870828693386971	0.7788007830714049
6	0.7071067811865476	-1.217940733217191	1.554859717121216	-1.80889999717125	0.7408182206817179
7	0.7071067811865476	-1.211136595042793	1.528727005901769	-1.74783261354842	0.7046880897187134
8	0.7071067811865476	-1.204332456868396	1.502740696425848	-1.687621730716286	0.670320046035639

CLIM file

```
#------------------------------------------------------------------------------------
  TITLE        Multi-trait RR TD model
  DATAFILE     /homeappl/home/ejo31/MiX99_course_Italy2025/9traits_TDdata/data/TD.data
  PEDFILE      /homeappl/home/ejo31/MiX99_course_Italy2025/9traits_TDdata/data/TD.pedi
  TABLEFILE    /homeappl/home/ejo31/MiX99_course_Italy2025/9traits_TDdata/data/LG3_W004.cov
  PARFILE      covarcomp.para
  RESIDFILE    residual.para
  INTEGER      HERD ANI TGRP HTM HC2Y C2Y AGE DCC DDRY DIM ResCL
  REAL         MILK PROTEIN FAT
  MISSING      -64.0
  DATASORT     BLOCK=HERD PEDIGREECODE=ANI
  TRAITGROUP   TGRP
  TABLEINDEX   DIM
  RESIDUAL     ResCL
  MODEL
   MILK(1)    =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   PROTEIN(1) =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   FAT(1)     =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   MILK(2)    =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   PROTEIN(2) =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   FAT(2)     =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   MILK(3)    =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   PROTEIN(3) =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
   FAT(3)     =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y HTM  HRDcurve(t2 t3 t5 |HC2Y)  AE(1 t2 t3 t5|ANI)  G(1 t2 t3 t5|ANI)
  RANDOM       HTM  HRDcurve  AE  G
  PEDIGREE     G  am
  WITHINBLOCKORDER G  AE  HTM  HRDcurve
  PRECON            b  b   b    b          b
  TMPDIR       tmpMiX
#------------------------------------------------------------------------------------
```

For this model we have scaled the observations milk(1)/1.6, protein(1)/0.07, fat(1)/0.11, milk(2)/1.9, …
It may improve:
- variance component estimation
- building of covariance functions
- solving the model

VC parameter file

1	1	1	0.4132767
1	2	1	0.3074280
1	2	2	0.3033726
1	3	1	0.2009712
1	3	2	0.1836562
1	3	3	0.2559136
1	4	1	0.3498057
1	4	2	0.2608165
1	4	3	0.1597769
1	4	4	0.3334821
1	5	1	0.2323996
1	5	2	0.2339328
1	5	3	0.1368069
1	5	4	0.2252217
1	5	5	0.2054421
1	6	1	0.1527369
1	6	2	0.1463706
1	6	3	0.1948430
1	6	4	0.1395105
1	6	5	0.1243335
1	6	6	0.1702153
1	7	1	0.3163034
1	7	2	0.2410058
1	7	3	0.1421130
1	7	4	0.3026225
1	7	5	0.2082898
1	7	6	0.1233145
1	7	7	0.2860063
1	8	1	0.1928116
1	8	2	0.2005343

Multi-trait covariance function model

Data file

```
ANI  HRD TGRP HTM HC2Y C2Y AGE DCC DRY DIM RC milk  protein  fat
2669 464 2 128223 266903 3 6 1 3 309 11 10.9  8.0  6.8
2669 464 2 128224 266903 3 6 1 3 335 12  9.2  7.0  6.0
2669 464 3 128215 266904 4 7 1 4  33  2 20.7 12.4  9.9
2669 464 3 128216 266904 4 7 1 4  73  5 17.9 10.4  8.6
```

Table file

This model requires 599 different covariates

```
DIM t1                 t2                  t3                 t4                  t5 ....................... t599
5 0.7071067811865476 -1.224744871391589 1.58113883008419 -1.870828693386971 0.7788007830714049
6 0.7071067811865476 -1.217940733217191 1.554859717121216 -1.808899997017125 0.7408182206817179
7 0.7071067811865476 -1.211136595042793 1.528727005901769 -1.747832613541842 0.7046880897187134
8 0.7071067811865476 -1.204332456868396 1.502740696425848 -1.687621730716286 0.670320046035639
```

CLIM file

covariance functions

```
#------------------------------------------------------------------------------------------------------------------
  TITLE        Multi-trait covariance function TD model with reduced parameters
  DATAFILE     /homeappl/home/ejo31/MiX99_course_Italy2025/9traits_TDdata/data/TD.data
  PEDFILE      /homeappl/home/ejo31/MiX99_course_Italy2025/9traits_TDdata/data/TD.pedi
  TABLEFILE    CovariableTable.txt
  PARFILE      Parfile_reduced.txt
  RESIDFILE    residual.para
  INTEGER      HERD ANI TGRP HTM HC2Y C2Y AGE DCC DDRY DIM ResCL
  REAL         MILK PROTEIN FAT
  MISSING      -64.0
  DATASORT     BLOCK=HERD PEDIGREECODE=ANI
  TRAITGROUP   TGRP
  TABLEINDEX   DIM
  RESIDUAL     ResCL
  MODEL
   MILK(1)    =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG( t6:11|HTM)@1  HRDcurve(  t60:74|HC2Y)@1  AE(t195:217|ANI)@1  G(t402:423|ANI)@1
   PROTEIN(1) =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t12:17|HTM)@1  HRDcurve(  t75:89|HC2Y)@1  AE(t218:240|ANI)@1  G(t424:445|ANI)@1
   FAT(1)     =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t18:23|HTM)@1  HRDcurve( t90:104|HC2Y)@1  AE(t241:263|ANI)@1  G(t446:467|ANI)@1
   MILK(2)    =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t24:29|HTM)@1  HRDcurve(t105:119|HC2Y)@1  AE(t264:286|ANI)@1  G(t468:489|ANI)@1
   PROTEIN(2) =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t30:35|HTM)@1  HRDcurve(t120:134|HC2Y)@1  AE(t287:309|ANI)@1  G(t490:511|ANI)@1
   FAT(2)     =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t36:41|HTM)@1  HRDcurve(t135:149|HC2Y)@1  AE(t310:332|ANI)@1  G(t512:533|ANI)@1
   MILK(3)    =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t42:47|HTM)@1  HRDcurve(t150:164|HC2Y)@1  AE(t333:355|ANI)@1  G(t534:555|ANI)@1
   PROTEIN(3) =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t48:53|HTM)@1  HRDcurve(t165:179|HC2Y)@1  AE(t356:378|ANI)@1  G(t556:577|ANI)@1
   FAT(3)     =  LACcurve(t1 t2 t3 t4 t5|C2Y) AGE DCC HC2Y  CG(t54:59|HTM)@1  HRDcurve(t180:194|HC2Y)@1  AE(t379:401|ANI)@1  G(t578:599|ANI)@1
  RANDOM       HTM  HRDcurve  AE  G
  PEDIGREE     G  am
  WITHINBLOCKORDER G  AE  CG  HRDcurve
  PRECON           b  b   b   b          b
  TMPDIR       tmpMiX
#------------------------------------------------------------------------------------------------------------------
```

combining effects across traits

VC file

```
1 1 1   1.0
1 2 2   1.0
1 3 3   1.0
1 4 4   1.0
1 5 5   1.0
1 6 6   1.0
2 1 1   1.0
2 2 2   1.0
2 3 3   1.0
2 4 4   1.0
2 5 5   1.0
2 6 6   1.0
2 7 7   1.0
2 8 8   1.0
2 9 9   1.0
2 10 10 1.0
2 11 11 1.0
2 12 12 1.0
2 13 13 1.0
2 14 14 1.0
2 15 15 1.0
3 1 1   1.0
3 2 2   1.0
3 3 3   1.0
3 4 4   1.0
3 5 5   1.0
3 6 6   1.0
3 7 7   1.0
3 8 8   1.0
3 9 9   1.0
3 10 10 1.0
3 11 11 1.0
```

Different model and solving alternatives

3 different models

Equations per random effect level **Original model** **Covariance function model**

Random effect	Amount of explained total variance		
	100.0	99.5	99.0
Herd test-month	9	6	5
Herd curve	27	15	13
Nonhereditary animal effects	36	23	20
Additive genetic animal effects	36	22	19

2 different preconditioners

- **Alt 1**: Block-diagonal preconditioner for all effects

```
WITHINBLOCKORDER G  AE  CG  HRDcurve
PRECON           b  b   b   b         b
```

- **Alt 2**: Diagonal preconditioner for G and AE effect

```
WITHINBLOCKORDER G  AE  CG  HRDcurve
PRECON           d  d   b   b         b
```

Solving alternatives

	Original model	**Covariance function model**	
	Amount of explained total variance		
	100.0	**99.5**	**99.0**
Number of Equations (reduction)	1 907 343 -	1 191 629 -38%	1 030 665 -46%
Block-diagonal preconditioners for all effects			
Number of iterations	513	478	474
Solving time (mm:ss)	2:41	2:42	2:20
Size of preconditioner iteration file (MB) (reduction)	130 6.5TB -	52 -60%	39 -70%
Diagonal preconditioners for animal effects			
Number of iterations	>10 000	1861	1715
Solving time (mm:ss)	33:14	9:27	8:05
Size of preconditioner iteration file (MB) (reduction)	9.4 -93%	5.4 270GB -96%	4.6 -97%

Expected size of pre-conditioner iteration files in case of 1M cows with records and a pedigree with 1.5 M animals

Luke

Correlations between 305-d breeding values

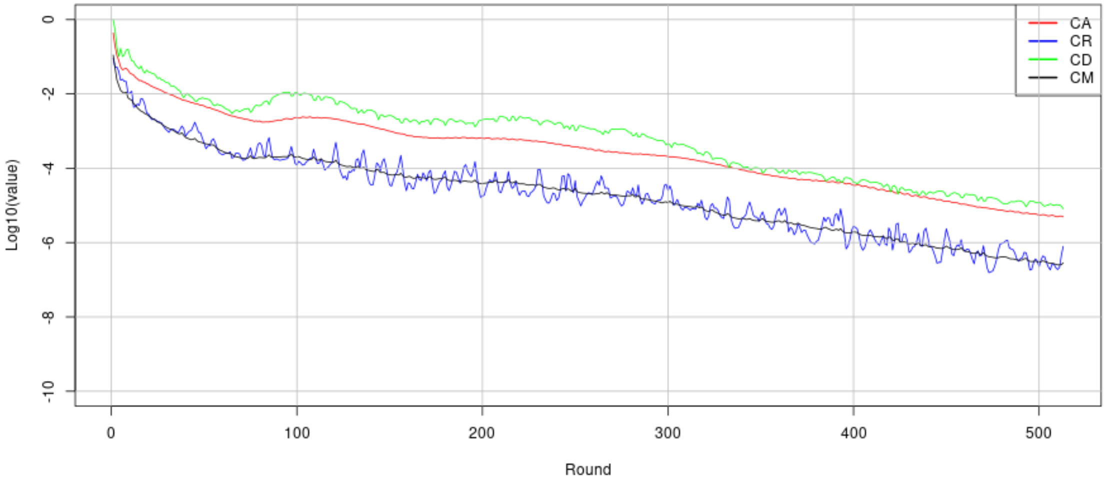
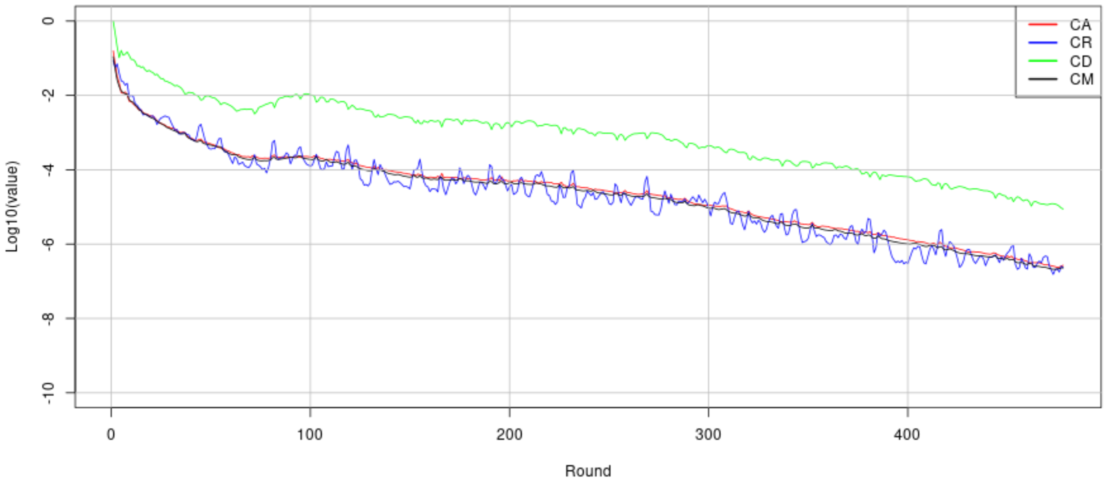
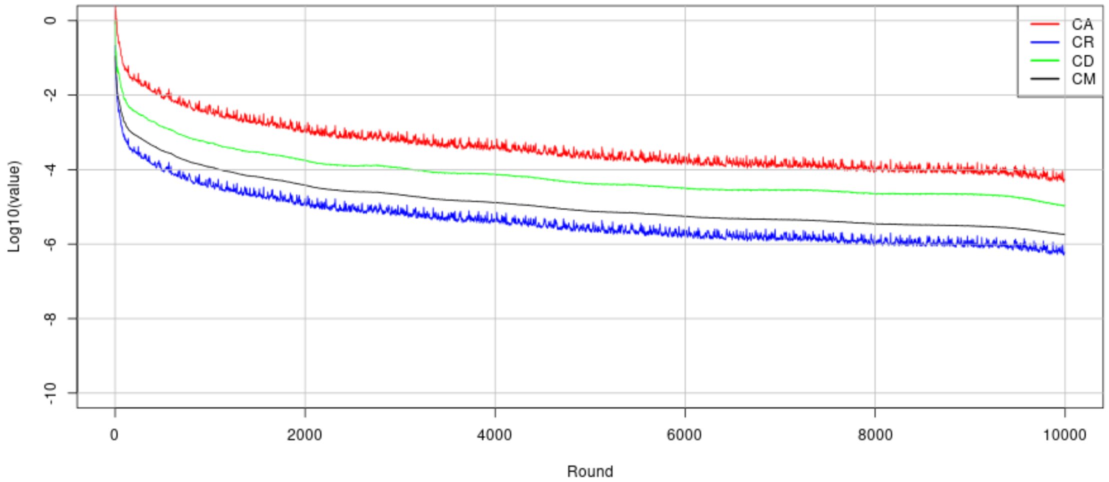
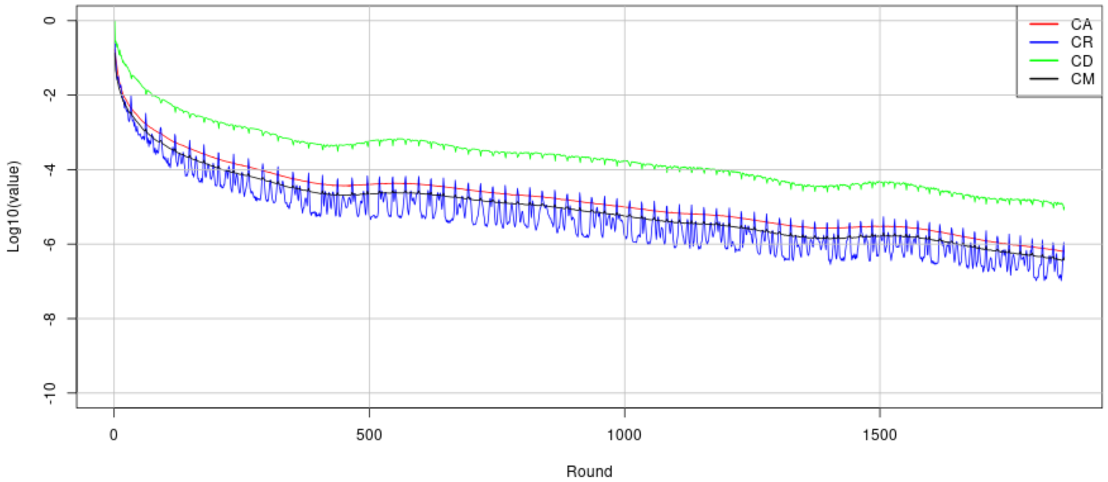
Original *versus* Covariance function 99.5%

	M1	P1	F1	M2	P2	F2	M3	P3	F3
M1	**0.995**	0.854	0.695	0.879	0.729	0.542	0.812	0.666	0.468
P1	0.842	**0.997**	0.796	0.737	0.878	0.659	0.669	0.802	0.560
F1	0.683	0.795	**0.997**	0.586	0.716	0.869	0.569	0.688	0.800
M2	0.872	0.735	0.589	**0.996**	0.844	0.678	0.980	0.838	0.656
P2	0.723	0.877	0.720	0.853	**0.996**	0.825	0.839	0.980	0.779
F2	0.530	0.649	0.867	0.680	0.815	**0.997**	0.715	0.847	0.982
M3	0.810	0.674	0.575	0.979	0.833	0.714	**0.997**	0.861	0.722
P3	0.657	0.800	0.688	0.839	0.976	0.850	0.859	**0.997**	0.838
F3	0.460	0.557	0.800	0.653	0.770	0.978	0.719	0.835	**0.997**

Original *versus* Covariance function 99.0%

	M1	P1	F1	M2	P2	F2	M3	P3	F3
M1	**0.993**	0.851	0.695	0.869	0.705	0.529	0.807	0.654	0.463
P1	0.831	**0.993**	0.792	0.742	0.872	0.663	0.670	0.799	0.559
F1	0.676	0.791	**0.994**	0.586	0.704	0.863	0.566	0.681	0.797
M2	0.871	0.731	0.588	**0.988**	0.818	0.664	0.978	0.827	0.653
P2	0.713	0.874	0.715	0.860	**0.988**	0.827	0.844	0.979	0.780
F2	0.524	0.645	0.864	0.681	0.802	**0.990**	0.717	0.843	0.981
M3	0.809	0.670	0.574	0.972	0.808	0.699	**0.991**	0.847	0.714
P3	0.649	0.797	0.684	0.846	0.968	0.852	0.861	**0.991**	0.836
F3	0.455	0.553	0.797	0.655	0.757	0.971	0.718	0.827	**0.992**

Convergence indicators CA, CR, CD, CM

Original model	Covariance function model 99.5%	Preconditioner
		Alt 1
		Alt 2

Thank you